Conics

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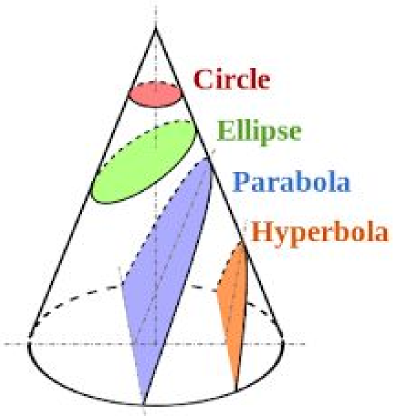
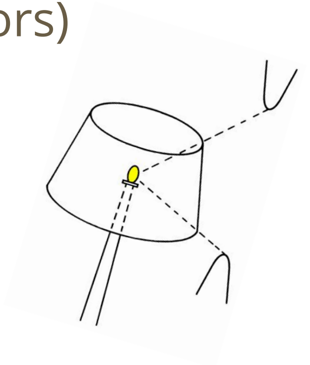
Exploring Conics (This is basically the summary too)

A conic section curve formed by intersection of a plane and double cone: by changing plane, one can create parabola, ellipse, circle or a hyperbola



Conics in real life

- **Why do conics matter?**
- Conic sections are very prevalent throughout our lives
- Many orbits are elliptical, our orbit around the sun is elliptical
- “Whispering galleries” are an interesting manifestation of this. If you whisper at one of the foci, you can hear it everywhere else in the gallery
- Parabolas describe the paths of projectiles
- Satellites use the properties of parabolas also (parabolic reflectors)
- Pizzas are circular, what more do you need really
- The light coming out of a cone shaped lamp makes a hyperbola



2.4 - Circles

A circle is a set of points in the xy-plane that are a fixed distance *r* (called the radius) from a fixed point (*h*, *k*) (called the center).

The **standard form of an equation of a circle** with radius *r* and center (*h*, *k*) is:

$(x-h)^2+(y-k)^2=r^2$

Setting it equal to y^2 leaves you with:

$(y-k)^2=r^2-(x-h)^2$

If the center of a circle is (0,0), then this equation simplifies to $x^2+y^2=r^2$, which is basically the Pythagorean Theorem all over again.

How to graph a Circle from an equation

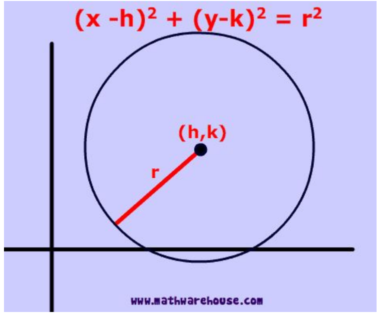
Change into y squared equals find what x can do to get zero. (EX: $X^2+Y^2=5$ ---> $Y^2= 5-x^2$).
Now, find what x values makes y = o. From there you will plug in values between those x values

Lines of symmetry: Every line that intersects the circle

- Remember square roots (so + and -)

Example: What is the standard form of the equation of a circle with radius of 2 and a center of (0,2)

~~You can do it~~



Example:

We start with the original equation

$(x-h)^2+(y-k)^2=r^2$

We substitute the values we know into the equation

$(x-(0))^2+(y-(2))^2=(2)^2$. We simplify and get our answer

$x^2+(y-2)^2=4$

Circles cont.

There's another equation we need to know, though, and it's referred to as the **general form of the equation of a circle**.

And here it is:

$x^2+y^2+ax+by+c=0$

We mostly use this equation to *graph circles.* This is done by simplifying the equation into quadratics. For example, what is the center and radius of a circle with the following equation?

$x^2+y^2-2x-4y-4=0$

Example

We start by moving the x terms and y terms together and moving the c to the right side, giving us

$x^2-2x+y^2-4y=4$

To complete the square of (x^2-2x), we take $(b/2)^2$:

$(2/2)^2 = 1^2 = 1$

So we add 1 to both sides, yielding

$(x^2-2x+1)+y^2-4y=5$

Next we complete the square for the y part.

Example cont.

We again use the formula $(b/2)^2$ to get our c term.

$((-4)/2)^2$

$= (-2)^2$

$= 4$

Since we got 4, we add 4 to both sides, yielding

$(x^2-2x+1)+(y^2-4y+4)=9$

Next, we will have to simplify the quadratics

Finishing it up

$(x^2-2x+1)+(y^2-4x+4)=9$

The first quadratic simplifies into $(x-1)^2$, and the second one simplifies into $(y-2)^2$. This leaves us with our standard form of the equation of the circle

$(x-1)^2+(y-2)^2=3^2$ (or 9, either way works)

To graph this circle, we would use the values of *h* and *k* (1 and 2, respectively) to move the center of the circle. Since *h*'s value is 1, we move the center one unit to the right and since the value of *k* is 2, we move the center 2 units up. Finally, since r^2 is 9, *r* is 3, so our circle is our set of all of the points that are 3 units away from the center (1,2).

Summary for 2.4

A circle is the set of all points that are *r* units from a fixed point *(h,k)*.

The **standard form of an equation of a circle** with radius *r* and center *(h, k)* is:

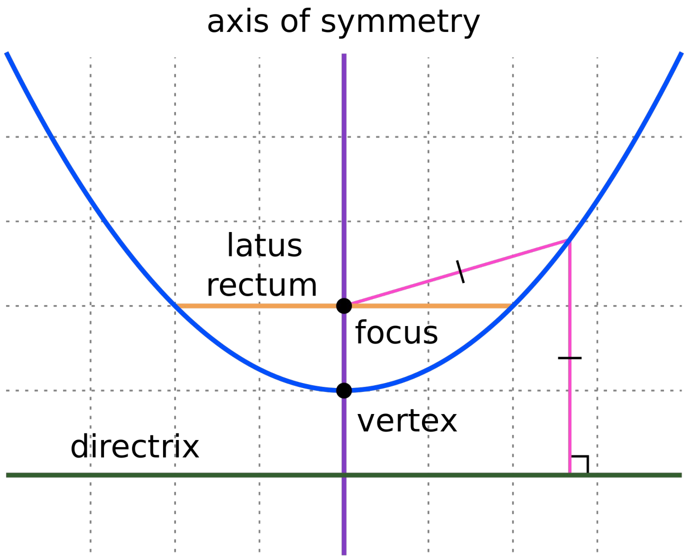
$(x-h)^2+(y-k)^2=r^2$

general form of the equation of a circle:

$x^2+y^2+ax+by+c=0$

Parabolas 11.2

- A parabola is the set of all points in a plane so that they are the same distance from a fixed point (the focus) and a line (the directrix)
- The vertex is exactly halfway between the focus and the directrix
- The latus rectum is a line that includes the focus and is four times the length of the distance between the focus and vertex



Writing an equation for a graph

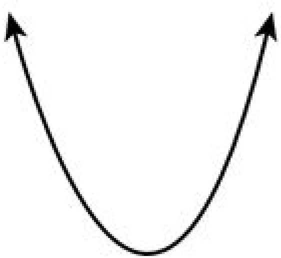
- You need to find all the points so that FP=PQ
  - F(0,3) focus
  - y=-3 directrix
  - (x,y) is a point on graph
- Write a distance formula from the focus to the point and another from the directrix to the point

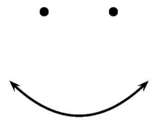
$$\sqrt{(x-0)^2+(y-3)^2} = \sqrt{(x-x)^2+(y-(-3))^2}$$

Equations of Parabolas $x^2=4ay$ edition

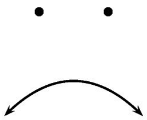
a>0

- Opens up
- Focus at (0,c)
- Directrix y=-c





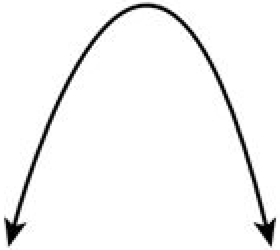
$a > 0$ (a positive smile)



$a < 0$ (a negative frown)

a<0

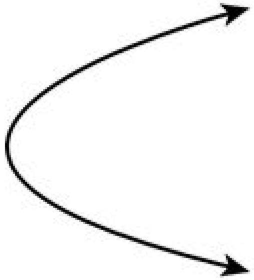
- Opens down
- Focus at (0, -c)
- Directrix y=c

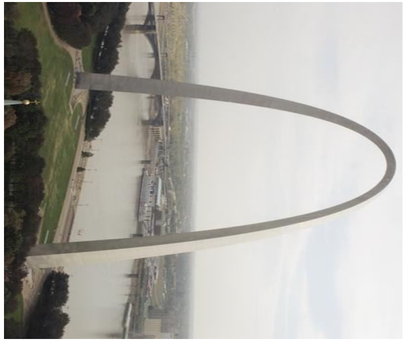


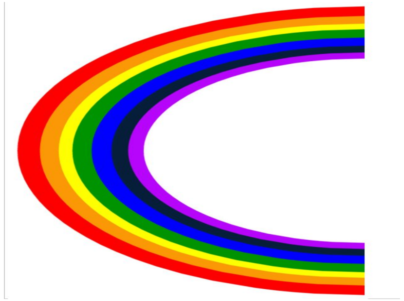
Equations of Parabolas, $y^2=4ax$

a>0

- Opens to the right
- Focus at (c,0)
- Directrix $x=-c$



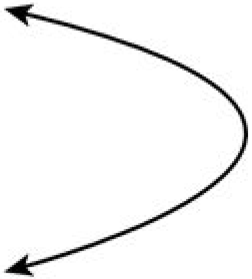




*Remember that *a* is the same thing as *c*

a<0

- Opens to the left
- Focus at (-c,0)
- Directrix $x=c$



Example

Identify the focus and directrix of the equation $y^2=16x$

1. Write the equation as $y^2=4ax$ to find a
   a. $y^2=(4)(4)x$ a=4

2. Because a>0, then the focus is (c,0) and the directrix is at x=-c

   a. Focus (4,0)

   b. Directrix $x=-4$

Identifying the vertex/translations

- Complete the square to put the equation into vertex form.
- This is also good for showing the translations

Vertex form for $x^2=4ay$ is $(x-h)^2=4a(y-k)$

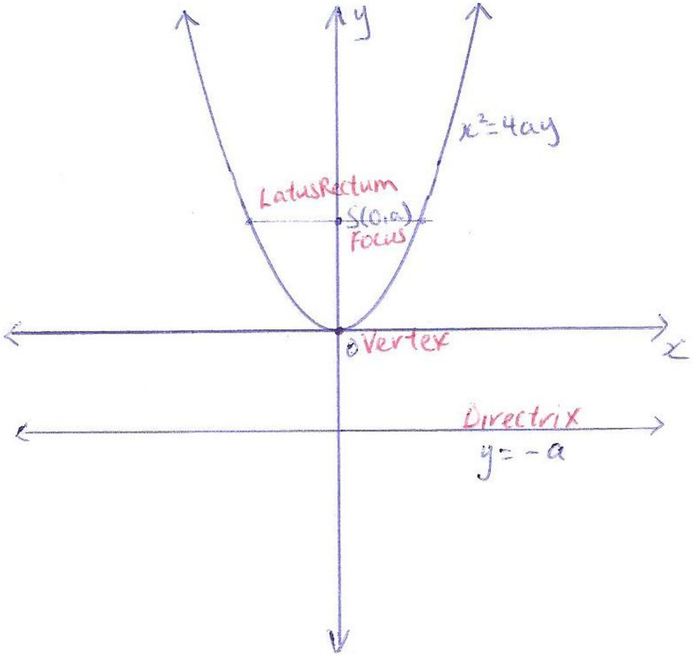
Vertex form for $y^2=4ax$ is $(y-k)^2=4a(x-h)$

- The vertex is at (h,k)
- Horizontal movement is shown in h
- Vertical movement is shown in k
- **The focus and directrix move with the vertex**

The latus rectum

The latus rectum is a line that

- has its endpoints on the parabola
- is parallel to the directrix
- Intersects the focus
- Is 4a units long



Example

Find the vertex and two points that define the latus rectum from the equation

$x^2-6x-4y+1=0$

1. Put into vertex form by completing the square
   a. $x^2-6x+(-3)^2=4y-1+(-3)^2$
   b. $(x-3)^2=4y+8$
   c. $(x-3)^2=4(y+2)$
   d. Vertex at $(3,-2)$
2. Find a
   a. $(x-3)^2=(4)(1)(y+2)$
   b. $a=1$

Example continued

1. Use a to find the focus
    a. a=1 focus at (0,1) --*but wait, remember how the vertex is at (3,-2)! Don't forget to translate it*!
    b. Focus after translation (3, -1)
2. Use the focus to find the latus rectum points
    a. Because our equation is a translation of the equation $x^2=4ay$, we need to use the y value of the focus, (if it were $y^2=4ax$, we'd use the x value)
    b. Plug in the y value (-1) and solve for x
        i. $(x-3)^2=4(-1+2)$
        ii. $x^2-6x+9=4$
        iii. $x^2-6x+5=0$
        iv. $(x-5)(x-1)=0$
        v. $x=5$ $x=1$
        vi. The points are (5,-1) and (1, -1)

Summary 11.2 (and graphing)

- We can use the vertex, the latus rectum and the focus to graph the equation
- The parent equations of parabolas are
  - $y^2=4ax$
  - $x^2=4ay$
- The translated equations, or vertex forms of parabolas are
  - $(y-k)^2=4a(x-h)$
  - $(x-h)^2=4a(y-k)$

11.3 Ellipses

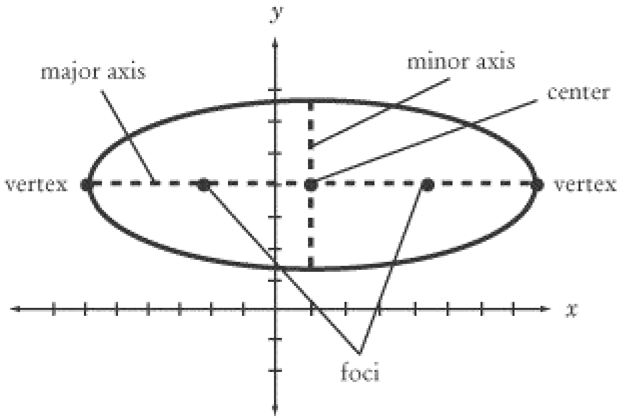
Definition: Set of points on a plane where sum of distances from P to two fixed points f1 to f2 is a constant.

PF1 + pF2 = K.

- Where F is a foci of the ellipse

Major axis: Contains the foci and has its endpoints on the ellipse
----Endpoint called **vertices**

Minor axis: Perpendicular to the major axis
----Endpoints called **co-vertices**



Ellipse Equations

When major axis is horizontal:
- **vertices:** (+/- a, 0) and **co-vertices** (0, +/-b)

When major axis is vertical:
- **vertices:**(0, +/-a) and **co-vertices:** (+/-b,0)

Don't forget h and k

Horizontal Major Axis

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

a>b

Vertical Major Axis

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

a>b

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Find C:
$C^2 = a^2 - b^2$

Find Foci:
Horizontal Major axis: (+/- c, 0)
Vertical Major axis: (0, +/- c)

Ellipse

Ellipse example:

center --(-3,-2), the major vertical axis of length 8(a) , the minor axis of length 6 (b)

Divide the distance by 2: a= 4, b=3
Equations: (h,k) $(x-h)^2/b^2 + (y-k)^2/a^2$

$(x+3)^2/9 + (Y+2)^2/16=1$

Example

Find the center, define the major and minor axises and graph the ellipse from the equation $\frac{(x+3)^2}{9}+\frac{(y-5)^2}{3}=1$

1. h=-3 k=5 a=3 b=√3
2. center: (-3,5)
3. Major axis defined by (-6,5) and (0,5)

Summary 11.3

When major axis is horizontal:
- **vertices:** (+/- a, 0) and **co-vertices** (0, +/-b)

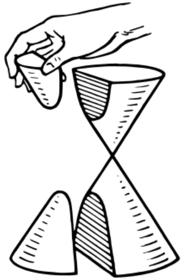
When major axis is vertical:
- **vertices:**(0, +/-a) and **co-vertices:** (+/-b,0)

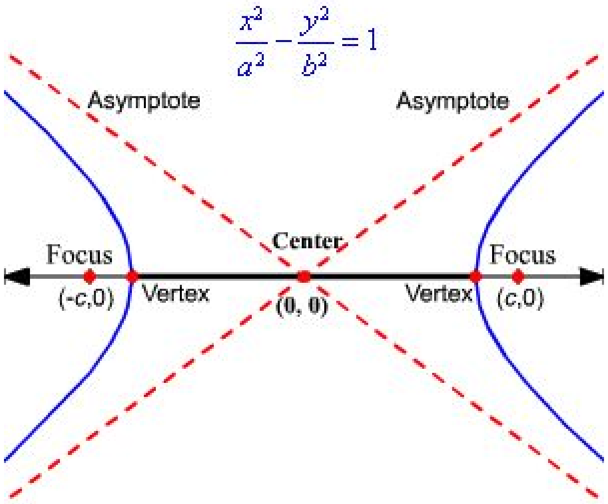
Horizontal Major Axis	Vertical Major Axis
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$
a>b	a>b

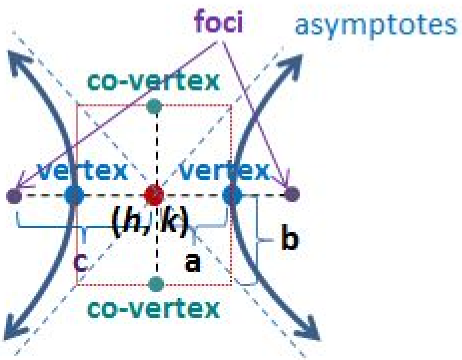
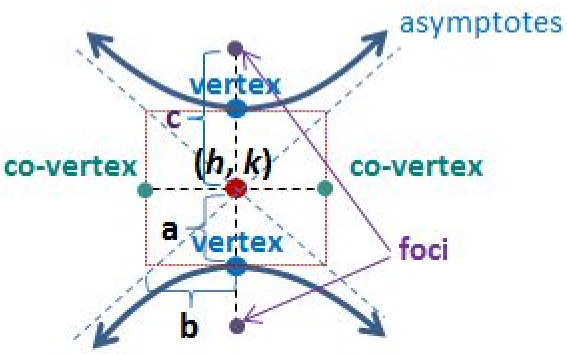
Find Foci:
Horizontal Major axis: (+/- c, 0)
Vertical Major axis: (0, +/- c)

Hyperbolas 11.4

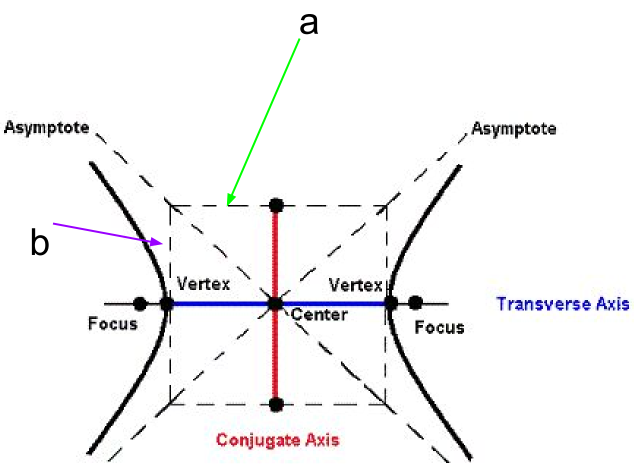
- Hyperbola is a set of points so that the absolute value of the difference between the distances from P to 2 fixed points (F_1 and F_2)
- Get pic of the equation formula thingy
- F_1 and F_2 are the foci of the hyperbola
- The line that contains the foci is the **transverse axis**
    - This is helpful for identifying hyperbolas.





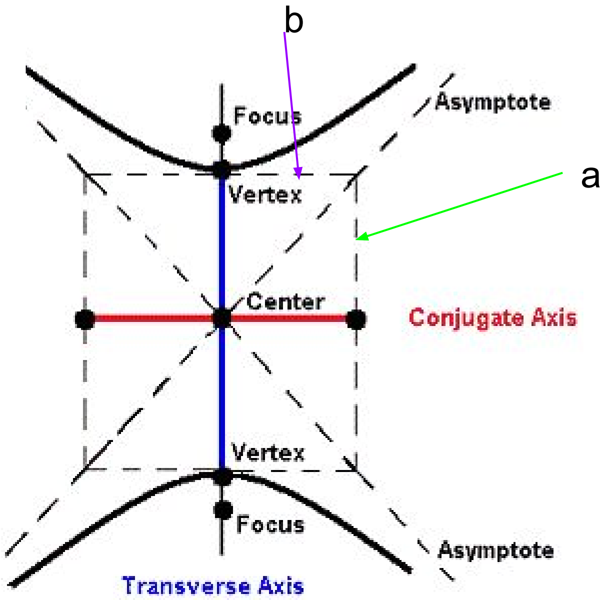
Horizontal Hyperbola (x^2 comes first)	Vertical Hyperbola (y^2 comes first)
At (0, 0): $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	At (0, 0): $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$
General: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	**General:** $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$
$a^2+b^2=c^2$	$a^2+b^2=c^2$
Center: (h,k) **Foci:** $(h\pm c,k)$	**Center:** (h,k) **Foci:** $(h,k\pm c)$
Vertices: $(h\pm a,k)$ **Co-Vertices:** $(h,k\pm b)$	**Vertices:** $(h,k\pm a)$ **Co-Vertices:** $(h\pm b,k)$
Asymptotes: $y-k=\pm\frac{b}{a}(x-h)$	**Asymptotes:** $y-k=\pm\frac{a}{b}(x-h)$
	

Examining the equation



Horizontal: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Vertical: $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

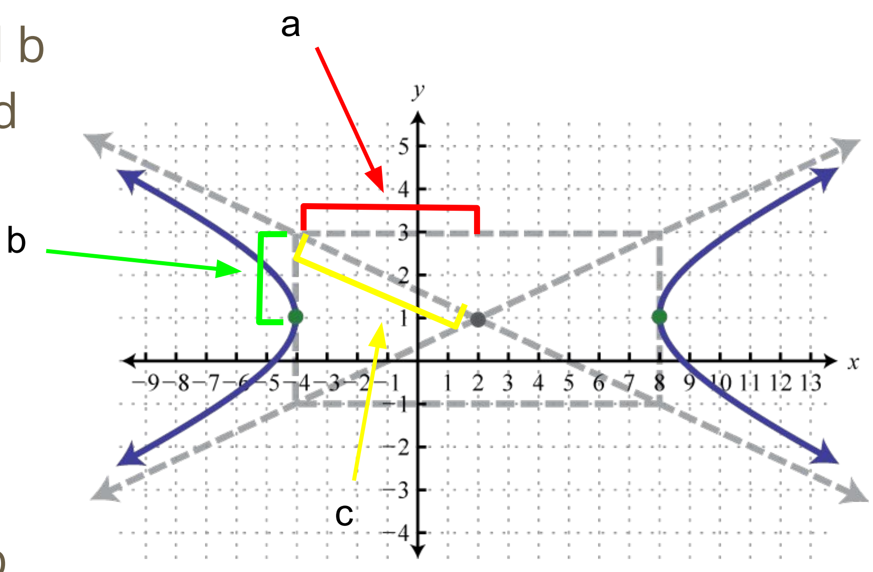


A is equal to the distance between the vertex and the center

B is equal to the distance between the vertex and the asymptote-more on that later

Using a, b, and c to graph hyperbolas

1. First put your equation into standard form
2. Use the standard form to find a and b
3. Use the pythagorean formula to find c
    a. This makes sense because of this
4. Use c to find foci--don't forget translations!!
5. Graph foci and vertices (using a)
6. Use a and b to make a rectangle so that the rectangles sides are 2a x 2b
7. Connect the corners to make the asymptotes



Example-turning equation into standard form

putting equations into standard form.

standard form: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $\frac{y^2}{a^2} - \frac{x^2}{b^2}$

~~Graph [illegible]~~

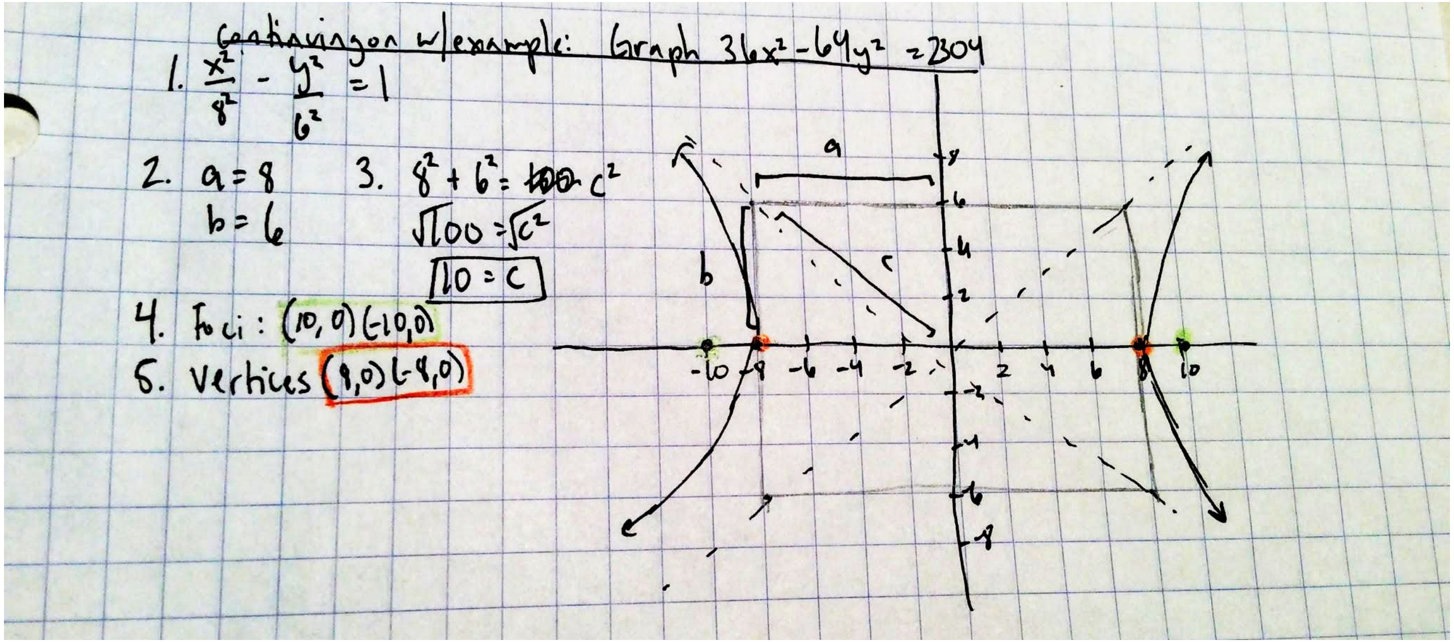
Graph $36x^2 - 64y^2 = 2304$

① $$\frac{6^2x^2 - 8^2y^2 = 2304}{6^2 \cdot 8^2}$$

② $$\frac{\cancel{6^2}x^2}{\cancel{6^2}\cdot 8^2} - \frac{\cancel{8^2}y^2}{6^2\cdot \cancel{8^2}} = \frac{\cancel{2304}}{\cancel{6^2\cdot 8^2}}$$

voilà! $\frac{x^2}{8^2} - \frac{y^2}{6^2} = 1$

Example-steps 2-7



11.4 Hyperbolas Summary

The standard form for Hyperbolas is →

Horizontal: $$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Vertical: $$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

The two important things to remember when graphing are 1. Put the equation into standard form 2. Find a,b,c

The line that contains the foci is the transverse axis. This determines whether it is Horizontal or Vertical.

Translating Conic Sections

This is a chart for all the types of graphs!

It's fantastic.

To proceed from here you basically plug in the number

And they hold true for the equations we are used to.

Just remember that h regulates horizontal movement and k regulates vertical movement

Summary — Families of Conic Sections

Conic Section	Standard Form of Equation		
Parabola	Vertex (0, 0) $y = ax^2$ $x = ay^2$	Vertex (h, k) $y - k = a(x - h)^2$ or $y = a(x - h)^2 + k$ $x - h = a(y - k)^2$ or $x = a(y - k)^2 + h$	
Circle	Center (0, 0) $x^2 + y^2 = r^2$	Center (h, k) $(x - h)^2 + (y - k)^2 = r^2$	
Ellipse	Center (0, 0) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	Center (h, k) $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$	
Hyperbola	Center (0, 0) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	Center (h, k) $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	horizontal vertical

Example-Circles

Write an equation for a circle with a center at (3,4) and a radius of 4 units

Use the center to find h&k $h=3$ $k=4$

Plug values into the equation: $(x-h)^2+(y-k)^2=r^2$

$(x-3)^2+(y-4)^2=4^2$

$(x-3)^2+(y-4)^2=16$ ---standard form

$x^2-6x+9+y^2-8y+16-16=0$

$x^2+y^2-6x-8y+9=0$---general form

Example-Parabolas

Write an equation for a parabola that has been translated left 5 and up 3 and whose parent equation is $y^2=24x$

Use the translation thing to find h and k $h=-5$ $k=3$

Use the equation $y^2=4ax$ to find a $y^2=(4)(6)x$, $a=6$

Plug h, k and a into the equation $(y-k)^2=4a(x-h)$ to find equation $(y-3)^2=4(6)(x+5)$

And simplify $(y-3)^2=24(x+5)$

Example-hyperbolas

Write an equation for a hyperbola with vertices at (2,-1) and (2,7) and foci (2,10) and (2, -4) Find a and b

A = distance between the vertices/2: 8/2, a=4

c= distance between foci/2 14/2, c=7

Use pythagorean theorem to get b= 33

Find center (3,-2) ----> equation $(y-3)^2/16 - (x+2)^2/33 = 1$

Translating Conics’ Summary

Summary — **Families of Conic Sections**

Conic Section	Standard Form of Equation	
Parabola	Vertex $(0, 0)$ $y = ax^2$ $x = ay^2$	Vertex (h, k) $y - k = a(x - h)^2$ or $y = a(x - h)^2 + k$ $x - h = a(y - k)^2$ or $x = a(y - k)^2 + h$
Circle	Center $(0, 0)$ $x^2 + y^2 = r^2$	Center (h, k) $(x - h)^2 + (y - k)^2 = r^2$
Ellipse	Center $(0, 0)$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	Center (h, k) $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$ $\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1$
Hyperbola	Center $(0, 0)$ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	Center (h, k) $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$ horizontal $\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$ vertical