Polynomials--From http://tutorial.math.lamar.edu/Classes/Alg/Polynomials.aspx

In this section we will start looking at polynomials. Polynomials will show up in pretty much every section of every chapter in the remainder of this material and so it is important that you understand them.

We will start off with **polynomials in one variable**. Polynomials in one variable are algebraic expressions that consist of terms in the form ax^n where n is a non-negative (*i.e.* positive or zero) integer and a is a real number and is called the **coefficient** of the term. The **degree** of a polynomial in one variable is the largest exponent in the polynomial.

Note that we will often drop the "in one variable" part and just say polynomial.

Here are examples of polynomials and their degrees.

$5x^{12} - 2x^6 + x^5 - 19$	Degree: 12
$x^4 - x^3 + x^2 - x + 1$	Degree: 4
$56x^{23}$	Degree: 23
$5x - 7$	Degree: 1
-8	Degree: 0

So, a polynomial doesn't have to contain all powers of x as we see in the first example. Also, polynomials can consist of a single term as we see in the third and fifth example.

We should probably discuss the final example a little more. This really is a polynomial even it may not look like one. Remember that a polynomial is any algebraic expression that consists of terms in the form ax^n. Another way to write the last example is $-8x^0$

Written in this way makes it clear that the exponent on the x is a zero (this also explains the degree…) and so we can see that it really is a polynomial in one variable.

Here are some examples of things that aren't polynomials.

$$5\sqrt{x} - x + x^2$$

$$\frac{2}{x} + x^3 - 2$$

It is important to remember that polynomials have UNRESTRICTED DOMAINS

The first example is not a polynomial because the domain is restricted to $(0, \infty)$. As a general rule of thumb if an algebraic expression has a radical in it then it isn't a polynomial.

The second example is not a polynomial again, because the domain is restricted to $(-\infty, 0) \cup (0, \infty)$ Another rule of thumb is if there are any variables in the denominator of a fraction then the algebraic expression isn't a polynomial.

Note that this doesn't mean that radicals and fractions aren't allowed in polynomials. They just can't involve the variables. For instance, the following is a polynomial

$$\sqrt[3]{5}\, x^4 - \frac{7}{12} x^2 + \frac{1}{\sqrt{8}} x - 5 \sqrt[14]{11}$$

There are lots of radicals and fractions in this algebraic expression, but the denominators of the fractions are only numbers and the radicands of each radical are only a numbers. Each *x* in the algebraic expression appears in the numerator and the exponent is a positive (or zero) integer. Therefore this is a polynomial.